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THE ATMOSPHERIC ABSORPTION CURVES AND THEIR DEPENDENCE ON THE NATURE OF THE PRIMARY COSMIC RAYS

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ABSTRACT. With a view to testing the possibility of interpreting the observed results on the absorption of cosmic rays in the atmosphere in terms of incoming electrons or positrons, the nature of the absorption curves produced by soft primaries has been calculated. Two different hypotheses have been assumed for the energy spectra of the primary, *viz*, (*a*) that the number of particles having energy E varies as $E^{-(\delta+1)}$ and (*b*) that only discrete sets of isoenergetic particles, produced through the annihilation of different atoms which are found in abundance in the interstellar space exist. The theoretical curves when compared with the observed Bangalore-Peshwar difference curve show that near the top of the atmosphere a good deal of difference in the absorption co-efficient between theory and observation exists. In this region however, the number of counts is much less than the actual ionization and the observations are also inaccurate.

A comparison of the absorption coefficient at large depths and also the sea-level latitude effect indicate definitely that the primary cosmic rays *must* contain protons. Whether electrons exist at all in the primary and if so whether hypothesis (*a*) or (*b*) approaches reality can only be determined by further observations made at closer intervals particularly, between 0° and 20°N. The observational results at Agra and Bangalore suggest, however, that even the protons in the primary cannot have a continuous energy spectrum

An application of the results of the cascade theory of showers can be made in the theoretical interpretation of the observed absorption curves in the atmosphere specially at high altitudes. Recent developments of cosmic-ray theory and experiments have made it possible to get deeper into the problem of the energy spectra of the various sorts of rays and the nature as well as the spectra of their primary. The high altitude measurements of the cosmic-ray intensity at different latitudes have been made by several authors [Bowen, Millikan and Neher (1938), Millikan, Neher and Pickering (1942), Neher and Pickering (1942)]. On the other hand the latitude effect at sea-level was also observed by Gill (1939), Compton and Turner (1937) and also by Millikan and Neher (1935). All these results show definitely that the cosmic-ray intensity is not the same at different places and is smallest for places near the equator. This is known as the latitude effect of cosmic radiation. The difference curves obtained by Millikan and others must all be produced by *charged* primary cosmic rays whose individual energies lie between definite limits depending on the geomagnetic latitudes of the places of observation and are given by the results of Störmer's theory as developed by Lemaitre and Vallarta. It is observed that all these difference curves have the same general shape and are in several respects similar to the theoretical absorption curves of the cascade theory. One is, therefore, tempted to conclude that these observed absorption curves are due to shower-formation by charged primary particles (electrons and positrons) having energies lying between certain well-defined limits. From the results of their observation Bowen, Millikan and Neher have derived a form for the primary energy spectrum incident at the top of the atmosphere. Naturally, from the atmospheric absorption curves measured at a small number of stations one cannot expect to derive a continuous spectrum for the primary rays. The curve derived by Bowen, Millikan and Neher is only a plausible one drawn in order to fit the observed results. The theoretical difference curves as calculated with the results of Bhabha and Heitler (1937) and those of Carlson and Oppenheimer (1937) when compared with the observed difference curves of Bowen, Millikan and Neher (1937) gave a qualitative agreement, but there was quite an appreciable amount of systematic difference between observation and theory. The maxima of the observed curves occurred at depths much less than the calculated ones. At the present time several authors are inclined to believe that up to date evidence is quite good for interpreting the observed results in terms of incoming protons *instead of* electrons. The main reasons they put forward are that the rate of rise of the ionization in the first metre of water of the atmosphere is a good deal more rapid than they can account for, by the cascade theory. Moreover the number of electrons which are usually obtained at depths of two or three metres of water below the top is according to them very much less than would be required by the cascade theory if the incoming rays are electrons. In view of the qualitative nature of the results obtained by Bhabha and Heitler and also by Carlson and Oppenheimer, and specially due to their inadequate treatment of the collision-loss it was difficult to say whether the above-mentioned differences were really due to the rough approximations made in the theory. The results obtained by Serber (1938) which have lately been used for theoretical calculations, when compared with those obtained by Bhabha and the present author (Bhabha and Chakrabarty, 1942) suggest that at least two of the above-mentioned difficulties may possibly be removed when the accurate results of the cascade theory are used. It is the purpose of the present paper to examine whether calculations based on the accurate results of the cascade theory can be compared with the observed curves and thereby to test whether the results of observation can be interpreted in terms of incoming electrons and positrons. Such a comparison will give an idea as to the nature and history of the primary cosmic rays before they enter the earth's atmosphere, which possibly bear the impress of the origin of cosmic rays.

The observational data so far obtained relating to the variation of the intensity of cosmic rays with altitude at different latitudes were not suitable for comparison with the theoretical results. When the measuring instruments are electroscopes or single counters which respond to rays reaching them from all directions instead of merely from the vertical, the analysis becomes much more complicated, especially in the equatorial latitudes. For comparison with the theoretical curves which give results for vertically incident primaries, the previous authors have applied Gross-transformation to the observed results or the inverse of the Gross-transformation to the theoretical results. But the uniformity of the primary radiation in azimuth, assumed in the derivation of the Gross equation does not really exist and the non-uniformity increases as one approaches the equator. This will be evident from the variation of the allowed cone with latitude, obtained by Lemaitre and Vallarta (1936). A consequence of this property is that the electroscope or single counter data cannot be used for any fine structure analysis of the primary, since the difference curves cannot be then considered as due to charged primaries lying between *sharply defined limits*. To overcome this difficulty Neher and Pickering (1942) have, however, used two counters as a cosmic-ray telescope to record the radiation coming from a definite direction. It may be doubted wheather the counter data do actually give the intensity of the radiation and in order to obtain the intensity at any altitude electroscopes should be used. The difference that may arise between counter and electroscope records have been tested, and Neher and Pickering conclude from their results of observation that the counting rates at any place, after proper adjustments, can fairly accurately be taken as proportional to the intensity of the cosmic rays at the corresponding position. A similar conclusion was also arrived at previously by Korff, Curtiss and Astin (1938).

Vertical coincidence measurements have been made by Neher and Pickering* only at Bangalore, Agra and Peshwar. We shall, however, use these results for the purpose of comparing the theoretical results deduced in the present paper.

Two different hypotheses have been postulated regarding the nature of the energy spectra of primary charged particles which are assumed for the present to be electrons or positrons. The possibility of the existence of protons in the primary will, however, be discussed at the end. The first hypothesis is (*a*) that the primary particles can have all possible energies and according to previous authors [Blackett (1941), Hilbery (1941), Johnson (1938), Heitler (1937)], the spectra can be represented by the equation,

$$f(E_0)dE_0 = \delta I \cdot \frac{k^\delta}{E_0{}^{\delta+1}} dE_0 \qquad \ldots \quad (1)$$

where δ lies between 1 and 2 and $f(E_0)dE_0$, gives the number of particles having energies between E_0 and $E_0 + dE_0$. Following Johnson and Blackett we shall take for the primary energy spectrum the form given by (1) where $\delta = 1.87$. k is, however, a constant quantity of the dimensions of energy, and may be suitably adjusted to give a good fit of the observed data with those calculated. The form (1) is also analogous to the one assumed by Heisenberg and

* Bhabha has informed me in a private communication that the angle subtended by their counters was so large that even this cannot be called a vertical measurement. In fact Millikan's two and three counter telescopes, which Neher and Pickering have used, record rays which pass through at an angle of as much as 45° from the vertical

Euler (1938) for the energy-spectrum of the mesons. The nature of the primary spectrum obtained by Millikan and others, however, can be made to coincide with (1) for values of $E_0 \geq 7 \times 10^9$. *e.v.* [*cf.* Hilbery (1941)]. Since for the purpose of comparison, with the results of observation, made in this paper we are concerned mainly with values of $E_0 > 7 \times 10^9$. *e.v.*, the results obtained with (1) will be nearly the same as that obtained with a primary distribution given earlier by Millikan and others.

The second hypothesis regarding the nature of the primary cosmic rays is (*b*) that the primary spectrum is not continuous but has lines or bands in which the the energy of the particles lie.

Since $f(E_0)dE_0$ is the number of charged particles (electrons or positrons) having energies lying between E_0 and $E_0 + dE_0$ and incident vertically on the top of the atmosphere, the average number of particles $C(\alpha, t)$, say, produced at a depth t, in radiation units, below the top of the atmosphere and coming in the vertical direction is given by

$$C(\alpha, t) = \int f(E_0) N(y_0, t) dE_0 \tag{2}$$

where $N(y_0, t)$ is the average number of particles produced at a depth t by a primary particle of energy β exp. y_0, incident vertically on the top of the atmosphere. β represents the mean collision loss corresponding to the material in which the shower is produced. The function $N(y_0, t)$ has been deduced in a previous paper by Bhabha and the present author. The value of α, however, depends on the geomagnetic latitude of the place of observation and can be easily deduced from the Lemaitre-Vallarta function. Substituting the values of $N(y_0, t)$ and simplifying, we get

$$C(\alpha, t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \delta I. \left(\frac{k}{\alpha}\right)^{\delta} . \left(\frac{\alpha}{\beta}\right)^{S-1} . \left\{\frac{1}{g_0(S)}\right\}^{S-1}$$
$$\times \frac{1}{(S-1)} . \frac{1}{(\delta - S + 1)} . \frac{D-\lambda}{\mu - \lambda} e^{-\lambda t} . dS$$
$$= \delta I. \left(\frac{k}{\beta}\right)^{\delta} . \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} exp. \left\{(S - 1 - \delta) y_\alpha - \lambda t\right\}$$
$$\times \frac{1}{(S-1)(\delta + 1 - S)} \left\{\frac{1}{g_0(S)}\right\}^{S-1} . \frac{D-\lambda}{\mu - \lambda} . dS \quad \ldots \quad (3)$$

where
$$y_\alpha = \log(\alpha/\beta) \tag{4}$$

and λ, μ, $g_0(S)$, etc., are all functions of S and are given in a previous paper (Bhabha and Chakrabarty, 1942). The integral on the right side of (3) can be evaluated quite accurately by the saddle point method and the method of such evaluation has been given previously (Chakrabarty, 1942). But $C(\alpha, t)$ is not the quantity, which can be compared directly with the results of observation, since in the observed absorption curves in any particular latitude exists also the particles produced by the uncharged primaries and also by any other charged particles which are not electrons or positrons or do not multiply according to the cascade process. If α_1 and α_2 are the values of α at two different latitudes then $C(\alpha_2, t) - C(\alpha_1, t)$ will give the difference curve which should compare with the observed difference curve corresponding to the latitudes of the places of observations, which obviously represents the effect of the latitude sensitive part of the primary cosmic rays having energies lying between α_2 and α_1 if it consisted only electrons and positrons. In Table I are given the values of α and y for some different places of observations. The geomagnetic latitude of the places of observation are also given in the second row of the table.

TABLE I

Place of observation	Bangalore	Agra	Peshwar	San Antonio	Oklahoma City	Omaha	Bismark
Magnetic latitude	3°N	17° 3N	25°N	38° 5N	45°N	51°N	57°N
α in B e v.	15 30	12 80	11.39	7.55	4.3	2.9	1.39
y_α	5 001	4 820	4 706	4.294	3 732	3.338	2 602

The values of $C(\alpha, t)$ as obtained from (3) for different values of α (*i.e.*, of y_α), and t can be obtained from Table II where the values of $1.80 \times 10^5 . I^{-1} k^{-\delta} \beta^{\delta} . C(\alpha, t)$ have been given. These values give immediately the theoretical difference curve between any two given places of observations listed in Table I.

TABLE II

Values of $1.80 \times 10^5 . I^{-1} k^{-\delta} \beta^{\delta} . C(\alpha, t)$ for different values of y_α and t

y_α \ t	1	2	3	4	5	6	8	10	12	15
5.001	87 5	218 4	324 6	375 6	360.0	308.1	186.4	94.9	42.9	12.1
4.820	110 1	280.0	404.2	446.8	419.2	352 8	201.2	99.7	46.0	12.6
4 706	137 6	312 4	455.2	499.0	460 6	383.4	220.6	106.0	48.4	12.8
4 294	216 0	553 0	715.0	715 0	616 2	487 8	258 4	119.2	50.9	—
3.732	644.0	1148.0	1280.0	1130 0	891 2	672.4	317.6	136.2	58.0	—
3.338	1226.0	1845.0	1839.0	1530.0	1139 0	803.6	368.2	166.6	...	—

If Millikan's hypothesis be accepted then in view of the results of Bowen and Wise (1939) on the abundance of the atoms in space, the primary cosmic rays should mainly consist of five definite cosmic-ray bands, arising out of the complete transformation of helium, carbon, nitrogen, oxygen and silicon atoms into cosmic rays. In Table III are given the values of the energy of the primaries that may be obtained from different atoms, together with the lowest value of the latitude ϕ_0 at which such rays may be incident on the top of the earth's atmosphere in the *vertical* direction.

TABLE III

Element	He	C	N	O	Si
E_0 in B.e.v.	1.9	5.6	6.6	7.5	13.2
y_0	2.915	4.000	4.160	4.288	4.853
ϕ_0	54°N	13°N	40°N	38°N	16°N

Since the primary consists of discrete sets of isoenergetic particles, we have on this hypothesis

$$C(a, t) = \Sigma I(y_0) N(y_0, t) \tag{5}$$

where $I(y_0)$ is the intensity of the primary having energy β exp. y_0 and the summation is to be taken for all values of y_0 given in Table III for which $E_0 \geqq a$. Hence, as in the previous case,

$$C(a_2, t) - C(a_1, t) \tag{6}$$

will give the theoretical difference curves. The values of I in this case will be proportional to the relative abundance of the different elements in space. In Table IV we have given the values of $N(y_0, t)$ for different values of y_0 corresponding to the different elements. With equations (5) and (6) it will then be possible to get the theoretical difference curves for any pair of latitudes lying in the northern hemisphere or, better say, between 3°N to 54°N.

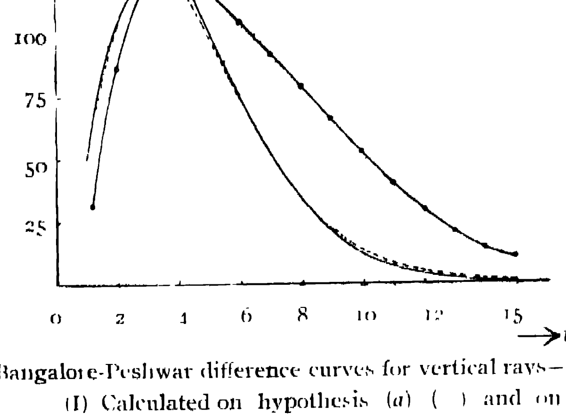
TABLE IV

Values of $N(y_0, t)$ for different values of y_0 and t

y_0 \											15
2.915	3.14	2.82	1.82	1.08	0.628	0.369	—	—	—	—
4.000	3.88	6.21	6.25	1.77	3.08	2.14	0.806	0.280	0.0947	0.0165
4.288	4.19	7.40	7.97	6.61	4.90	3.33	1.32	0.047	0.0154	0.0285
4.853	4.86	9.90	12.6	12.1	9.88	7.21	3.26	1.28	0.464	0.0894

One very important difference exists between the two hypotheses which will possibly be sufficient to determine whether the primary cosmic radiation has a continuous energy spectrum or it only consists of discrete sets as postulated in hypothesis (*b*). It is apparent from Tables I and II that on the hypothesis (*b*) no difference should exist between the vertical coincidence curves taken at (*i*) Peshwar and Agra, (*ii*) Oklahoma City and Omaha, and also possibly between (*iii*) Bismark and any other higher latitudes, but on the hypothesis (*a*) differences must exist in these cases also. The observations made at the first pair of stations, *viz.*, Peshwar and Agra, made by Neher and Pickering (1942) puts a strong evidence against the hypothesis (*b*) whereas their observations taken at Agra and Bangalore invalidates both the hypotheses. Similar observations should be made at other latitudes also before making any definite conclusion on this point. The theoretical difference curves for other latitudes can be easily obtained from Tables II and IV.

In Fig. 1 we have plotted the Bangalore-Peshwar difference curve as given by Neher and Pickering.*The theoretical curves as obtained for the two hypotheses have also been drawn. The ordinates of the theoretical curves are so adjusted (such adjustments will possibly give the values of k and I) that all the curves give the same maximum intensity. It will appear from the figure that the theoretical difference curves based on the hypothesis (*a*) and (*b*) are nearly identical so that a comparison of these curves only will give no indication as to the validity of either of the hypotheses. When compared with the observed curves it is significant that the observed rate of rise of the ionization in the first metre of water equivalent of the atmosphere is more rapid than the theoretical estimates based on the cascade theory. It may be noted, however, that in this region the observed data are possibly much less than the actual ionization and this is due to the use of counters instead of electroscopes for observations, since at these altitudes groups of particles must become increasingly prevalent and these register but once in the counter whereas they will give their true value in an electroscope. Such a difference between the counter and electroscope data at high altitudes has also been noticed by Neher and Pickering Hence to compare more accurately it is necessary to make observations with counter-controlled electroscopes, or any other device which will record the *true ionization produced only by vertical rays.*



Bangalore-Peshwar difference curves for vertical rays—
(I) Calculated on hypothesis (*a*) ( ) and on hypothesis (*b*) (- - - -)
(II) Observed curve (– o—o—)

FIG. 1

* For reasons mentioned above this cannot be compared with the theoretical curves. This has, however, been introduced to show qualitatively the order of the difference at large depths.

Figure 1 will also show that the discrepancy mentioned by previous authors as regards the position of the maximum disappears almost completely. Beyond the maximum the actual ionization is much more than can be explained by the cascade theory. Since these curves represent the ionization produced by primary particles in a definite range of energy the discrepancy in the apparent absorption co-efficient at large depths cannot be explained by the presence of a few primary particles of very high energy. Consequently, to account for the difference between the observed and theoretical estimates of the ionization at great depths it is necessary to postulate the creation of charged particles by processes other than the cascade process, for which the primary must be charged particles within definite range of energy. The excess ionization observed has been suggested by several authors as due to mesons and their decay particles. If such an explanation is to be accepted then it is necessary to postulate that mesons are somehow produced at least in part by charged particles. These results suggest that the primary cosmic rays should also consist of protons. The experiments of Schein, Jesse and Wollan (1941) lend support to this view.

Although the observed Agra-Bangalore difference curve cannot be explained by hypothesis (*a*) there exists a serious difficulty in accepting the hypothesis (*b*). The energy of some of the most energetic rays are known to be more than a thousand times the mass of the heaviest known atom and as such, hypothesis (*b*) cannot explain the existence of such particles. The validity of either of the two hypotheses can only be tested by making further observations at closer latitudes. If, however, the discrepancy mentioned above between the theoretical and the observed curves on the left of the maximum persists even with more accurately obtained observational results, it may be necessary to accept the view that electrons and positrons are not at all present in the primary cosmic rays, and the charged primary particles are entirely protons. In that case, however, in order to explain the observed facts it will be necessary to admit the existence of hitherto unknown process which allows a complete absorption of protons in the very upper layers of the atmosphere and a consequent production of mesons, electrons or γ-rays.

The latitude effect at sea-level has been observed by several authors. But from the results already published (Bhabha and Chakrabarty, 1942, Table III) it will be clear that even for a primary particle (electron or positron) with energy as high as 40. B.e.v, there is a very little probability of its effect being felt at sea level. Consequently the sea-level latitude effect cannot be produced by primary electrons or positrons, and requires the existence of protons in the primary cosmic rays. This conclusion will be altered if we assume that electrons can produce mesons or some other particles which require a lower energy to penetrate the atmosphere than by the cascade process.

It is therefore essential that protons or at least some charged particles other than electrons or positrons *must* exist in the primary cosmic rays. A similar conclusion was also arrived at by Johnson (1939) from other considerations. Whether electrons or positrons do exist at all in the primary cannot, however, be definitely established unless further observational data, taken at closer latitudes, are available. A few observations made at closer intervals between the equator and say 20° N will give definite indications as to the possibility of either of the two hypotheses and will also indicate whether electrons do exist in the primary. The results of the measurements of Neher and Pickering (1942) at Agra and Bangalore, however, suggest that even the protons in the primary cosmic rays cannot have a continuous energy spectrum.

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ERRATA

Dipole Moments of Palmitic Acid, Aleuritic Acid and Alkyl Esters of Aleuritic Acid—By G. N. Bhattacharya, *Ind. J. Phys.*, Vol. XVI

P. 372, line 7 from the bottom—

read ' p ' *instead of* ' P ' in the formula $P = \frac{\epsilon - 1}{\epsilon + 2} \cdot \frac{1}{d}$

P. 372, line 6 from the bottom—*read* ' ϵ ' *instead of* ' C '

P. 372, line 3 from the bottom—*read* $P_1 = M_1 \left(p_2 + \frac{p_{12} - p_2}{w} \right)$

instead of $P_1 = M_1 \left(P_2 + \frac{P_{12} - P_2}{w} \right)$

P. 372, line 2 from the bottom—*read* ' p_{12} and p_2 ' *instead of* ' P_{12} and P_2 '

P. 375, line 12 from the top—*read* ' carbon atom ' *instead of* ' carbonation '

P. 376, under reference 3—*read* ' Reinhold ' *instead of* Reihnold '

P. 376, under reference 18—*read* ' Swietonslawski ' *instead of* ' Swietoslwaski. '